Homework Week 5 - Computations not guaranteed

2.5 **3c** $Q = [id]_{\beta'}^{\beta}$ $1 = 1(3x^2+1) + (-3)x^2$

$x = (-1)(2x^2 - x) + 2(x^2)$

$x^2 = 1x^2$

$$Q = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ -3 & 2 & 1 \end{pmatrix}$$

4 $T : \mathbb{R}^2 \to \mathbb{R}^2$ $T(1,0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $T(0,1) = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$[T]_\beta = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$ $Q = [id]_{\beta'}^{\beta} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

$\therefore [T]_{\beta'} = Q^{-1}[T]_\beta Q = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ Do the matrix mult yourself

9 $A \sim A$ clear. Suppose $A \sim B$ $B = Q^{-1}AQ$

$\therefore QBQ^{-1} = A$ and $Q = (Q^{-1})^{-1}$ so $B \sim A$

Next suppose $A \sim B$, $B \sim C$ $B = Q^{-1}AQ$, $C = P^{-1}BP$

$\therefore C = P^{-1}Q^{-1}AQP = (QP)^{-1}AQP$ so $A \sim C$

10 $B = Q^{-1}AQ$

$Tr(B) = Tr(Q^{-1}(AQ)) = Tr((AQ)Q^{-1}) = Tr(A)$

11 (a) $Q = [id]_\alpha^\beta$, $R = [id]_\beta^\gamma$

$RQ = [id]_\beta^\gamma [id]_\alpha^\beta = [id]_\alpha^\gamma$ which is the change of coordinate matrix from α to γ

(b) $Q = [id]_\alpha^\beta$ $\therefore [id]_\beta^\alpha Q = [id]_\beta^\alpha [id]_\alpha^\beta = [id]_\alpha^\alpha = I$

$\therefore [id]_\beta^\alpha = Q^{-1}$

3.1 **2.** $\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \xrightarrow{2R_1 + R_3} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 7 \end{pmatrix} \xrightarrow{\frac{1}{7}R_3} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{-3R_3 + R_1} I_3$

3 (a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(c) $2R_1 + R_3$ applied to I_3 gives $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$

9. Suppose $R_i \longleftrightarrow R_j$ then apply the following row operations

$R_j + (-1)R_i$, $R_i + 1 \cdot R_j$, $R_j + (-1)R_i$, $(-1)R_j$

3.2 2 (a) C_2 (column 2) $= C_1 + C_3$ ∴ rank $= 2$

(c) rank ≤ 2 ∴ rank $= 2$

(f) $C_2 = 2C_1$ ∴ $-2C_1 + C_2$ and $C_2 \longleftrightarrow C_5$ give

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 3 & 0 \\ 3 & 1 & 2 & 8 & 0 \\ -4 & 1 & 1 & -3 & 0 \end{pmatrix} \xrightarrow{C_1 \longleftrightarrow C_2} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 3 & 0 \\ 1 & 3 & 2 & 5 & 0 \\ 1 & -4 & 1 & -3 & 0 \end{pmatrix} \xrightarrow[R_4 + (-1)R_1]{R_3 + (-1)R_1} \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & -5 & 1 & -4 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & -5 & 1 & -4 & 0 \end{pmatrix} \xrightarrow[\frac{5}{2}R_2 + R_4,\ \frac{1}{2}R_2]{-R_2 + R_3} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & \frac{7}{2} & \frac{7}{2} & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

rank 3

4 (a) $\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 0 & -1 & 2 \\ 1 & 1 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow$

$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ rank 2

5 (a) rank 2 $\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right) \xrightarrow{-R_1 + R_2} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right) \xrightarrow[-R_2]{2R_2 + R_1}$

$\left(\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right)$ inverse $= \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$

5 (b) rank 1, no inverse

6d First determine $[T]_\alpha^\beta$ where α, β are standard bases.

$$[T]_\alpha^\beta = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$ rank 3 : C_2, C_3 linearly independent, C_1 not linear comb. of C_2 and C_3

$\therefore$ matrix invertible $\therefore$ T invertible. Find inverse of matrix.

$$\underbrace{\left(\begin{matrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \end{matrix}\right.}_{A}\left|\left.\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right)\right. \xrightarrow[-R_1+R_3]{-R_1+R_2} \left(\begin{matrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & -1 & -1 \end{matrix}\right|\left.\begin{matrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{matrix}\right) \xrightarrow[\substack{R_2+R_1 \\ -R_2+R_3}]{\frac{1}{2}R_2}$$

$$\left(\begin{matrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{matrix}\right|\left.\begin{matrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{matrix}\right) \xrightarrow[\substack{-R_2 \\ -R_3}]{R_3+R_1} \left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right|\underbrace{\left.\begin{matrix} 0 & 0 & 1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & -1 \end{matrix}\right)}_{A^{-1}}$$

$T^{-1}(1) = (0, \frac{1}{2}, \frac{1}{2})$

$T^{-1}(x) = (0, -\frac{1}{2}, \frac{1}{2})$

$T^{-1}(x^2) = (1, 0, 1)$.

8 $L_{cA}(x) = (cA)x = c(Ax) = cL_A x = L_A(cx)$

If $y \in R(L_{cA})$: $y = L_{cA}(x)$ some x

$\therefore y = L_A(cx)$ $\therefore y \in R(L_A)$ so $R(L_{cA}) \subseteq R(L_A)$

Now $y \in R(L_A)$ $y = L_A(x)$ some x $\therefore y = L_A(c\frac{1}{c}x)$

$= L_{cA}(\frac{1}{c}x) \in R(L_{cA})$ $\therefore R(L_A) \subseteq R(L_{cA})$ $\therefore =$

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$$M(A|B) = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1n} \\ M_{21} & M_{22} & & M_{2n} \\ \vdots & & & \vdots \\ M_{m1} & M_{m2} & \cdots & M_{mn} \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & & a_{1r} & b_{11} & b_{12} & & b_{1s} \\ a_{21} & a_{22} & & a_{2r} & b_{21} & b_{22} & & b_{2s} \\ & & \vdots & & & & & \\ a_{n1} & a_{n2} & \cdots & a_{nr} & b_{n1} & b_{n2} & & b_{ns} \end{pmatrix} =$$

where M is $m \times n$, A $n \times r$, B $n \times r$, $M_1 \ldots M_m$ rows of M, $A_1, \ldots, A_r$ columns of A, $B_1, \ldots B_s$ columns of B

$$= \begin{pmatrix} M_1A_1 & \cdots & M_1A_r & M_1B_1 & \cdots & M_1B_s \\ M_2A_1 & & M_2A_r & M_2B_1 & \cdots & M_2B_s \\ M_mA_1 & & M_mA_r & M_mB_1 & \cdots & M_mB_s \end{pmatrix}$$